# DYNAMIC STABILIZATION OF THIN PLASMA FILAMENTS

M. G. Nikulin

The Routh function is found for a fine plasma filament (toroidal and rectilinear), experiencing smooth long-wave hosepipe and constrictive perturbations in the case in which the external magnetic field is a combination of quadrupole and longitudinal fields. Different variants of dynamic stabilization of a filament are briefly discussed. Combined dynamic stabilization of a straight filament with an alternating current by means of constant quadrupole and longitudinal fields is investigated by averaging with respect to rapid oscillations.

In [1,2] a method was developed for investigating the stability of fine plasma filaments relative to long-wave hosepipe and constrictive perturbations. The method was based on the fact that a plasma filament carrying a current can be treated as an electromechanical system for which the Routh function can be calculated, thus making it possible for the equation of motion to be obtained for the filament close to the equilibrium position. This method turns out to be particularly effective for solving problems of the dynamic stabilization of plasma filaments by quasi-stationary high-frequency magnetic fields. It was shown in [2] that the Routh function R for systems of this type can be written in the following form:

$$R = T - W. \tag{0.1}$$

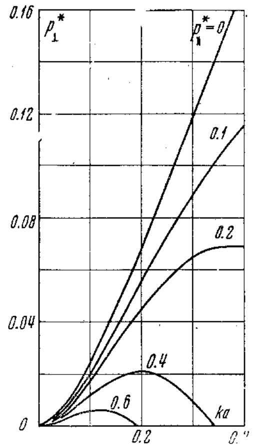


Fig. 1

Here T is the kinetic and W the generalized potential energy, which is equal to the sum of the internal U and magnetic self-energy $W_m$ of the system

$$W = U + W_m. \tag{0.2}$$

If the expressions for T and W are known, the potential energy averaged over the high-frequency oscillations of the motion can easily be obtained as a quadratic form with constant coefficients, and this can be investigated without difficulty. Using the method of averaging we can immediately evaluate this or that variant of dynamic stabilization. If resonance phenomena characteristic for systems with periodically varying parameters have to be investigated, the equations of motion obtainable with the help of the Routh function can be treated directly.

In the present paper we find the magnetic self-energy $W_m$ of a thin circular ring of plasma experiencing smooth long-wave constrictive and hosepipe perturbations for the case in which the external magnetic field is a combination of quadrupole and longitudinal fields. In addition to the mechanical Lagrangian ($L = T - U$), calculated in paper [1], the magnetic self-energy $W_m$ gives us the Routh function

$$R = L - W_m. \tag{0.3}$$

The instantaneous Routhian for an infinite straight filament is found by passing to the limit. The results obtained are used for a brief discussion of familiar methods of stabilizing kinks in a plasma filament: the use of quadrupole and longitudinal magnetic fields; the combined action of these fields on a straight filament carrying a high-frequency alternating current is also investigated (by the method of averaging).

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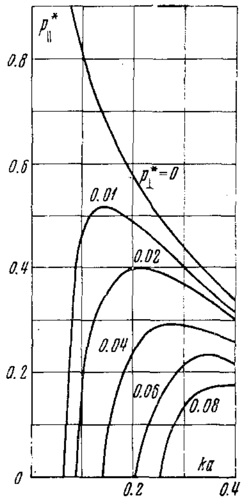


Fig. 2

1. Let us consider a thin circular plasma ring whose the smaller radius is $a$ and whose larger radius is b, $b \gg a$. The plasma is assumed to be ideally conducting, incompressible and nonviscous. The ring is situated in an external magnetic field having longitudinal and transverse components relative to the filament, The basic transverse field excites a longitudinal current in the ring and serves to stabilize the ring relative to its larger radius. In the neighborhood of the plasma filament there is also an additional transverse field having a quadrupole structure, which stabilizes the filament with respect to hosepipe perturbations. The magnetic fields and the currents which they induce may have high-frequency components.

Let us describe perturbations of the ring by the functions $\varepsilon(\varphi,t)$, $\delta(\varphi,t)$, and $\alpha(\varphi,t)$ which are small compared with unity:

$$\varepsilon, \delta, \alpha \ll 1, \qquad (1.1)$$

These enable the equations for the axial line and the smaller radius of the ring to be written in the form

$$r(\varphi, t) = b[1 + \varepsilon(\varphi, t)], \quad z(\varphi, t) = b\delta(\varphi, t), \quad a_{\sim}(\varphi, t) = a[1 + \alpha(\varphi, t)]. \qquad (1.2)$$

Here r, $\varphi$, and z are cylindrical coordinates of the axial line of the filament, $a_{\sim}$ is the variable radius of transverse cross section, perpendicular to the perturbed axis which is thus considered to be circular.

It is convenient to introduce a discrete description, expanding $\varepsilon$, $\delta$, and $\alpha$ in Fourier series of the form

$$\varepsilon(\varphi, t) = \varepsilon_0(t) + \sum_{n=1}^{\infty} [\varepsilon_{nc}(t)\cos n\varphi + \varepsilon_{ns}(t)\sin n\varphi]. \qquad (1.3)$$

We shall take the perturbations of the filament to be smooth, i.e., such that the displacement $\xi$ satisfies the inequality

$$k\xi_k \ll 1, \qquad (1.4)$$

where $k = n/b$ is the wave number of a given perturbation. Since $\xi \sim b\varepsilon$, $b\delta$, $a\alpha$, (1.4) may be written in the form

$$n\varepsilon_{nc\,(s)}, \quad n\delta_{nc\,(s)}, \quad n\frac{a}{b}\alpha_{nc\,(s)} \ll 1. \qquad (1.5)$$

Conditions (1.5) for $\varepsilon$ and $\delta$ are more restrictive than (1.1) and mean that the higher harmonics in expansions of type (1.3) have a vanishingly small weight. We shall also assume that kinks in the filament are "overdeveloped," i.e., $\xi \gg a$, or what is the same, $\varepsilon, \delta \gg a/b$. Thus in the case under consideration the kinks must satisfy the following inequalities

$$ka \ll k\xi_k \ll 1. \qquad (1.6)$$

We note that in the ordinary magnetohydrodynamic approach $k\xi_k \ll ka$, while $ka$ can be arbitrary.

Under the assumptions made above, the kinetic and internal energies are defined [1] by the expression

$$T = \frac{1}{4} M b^2 \left\{\left(\frac{a}{b}\right)^2 \dot{\alpha}_0^2 + 2(\dot{\varepsilon}_0^2 + \dot{\delta}_0^2) + \sum_{n=1}^{\infty}\left[\frac{4}{n^2}(\dot{\alpha}_n^2 + \dot{\alpha}_n\dot{\varepsilon}_n) + \left(1 + \frac{1}{n^2}\right)\dot{\varepsilon}_n^2 + \dot{\delta}_n^2\right]\right\} \qquad (1.8)$$

$$U = U^{(0)} - p_0 V^{(0)} [\langle 2\alpha + \varepsilon + 2\alpha\varepsilon + \alpha^2 + {}^1/_2(\varepsilon'^2 + \delta'^2)\rangle - {}^1/_2\,\gamma(2\alpha_0 + \varepsilon_0)^2]. \qquad (1.8)$$

Here

$$\dot{\alpha}_n^2 = \dot{\alpha}_{nc}^2 + \dot{\alpha}_{ns}^2, \quad \dot{\alpha}_n\dot{\varepsilon}_n = \dot{\alpha}_{nc}\dot{\varepsilon}_{nc} + \dot{\alpha}_{ns}\dot{\varepsilon}_{ns}.$$

$\dot{\varepsilon}_n^2$ and $\dot{\delta}_n^2$ have a structure similar to $\dot{\alpha}_n^2$; M is the mass of the ring; $p_0$ is the equilibrium gas pressure;

$\gamma$ is the adiabatic exponent; certain equilibrium quantities are denoted by the superscript (0), a dot in the prime position denotes differentiation with respect to time, the prime denotes differentiation with respect to $\varphi$, while angle brackets denote an average with respect to $\varphi$;

$$\langle f(\varphi)\rangle = \frac{1}{2\pi}\int_0^{2\pi} f(\varphi)\, d\varphi .$$

The functions T and U are obtained on the assumption that conditions are globally adiabatic, i.e., when the gas pressure is related to the total volume of the ring by the adiabatic equation.

2. Let us now find the magnetic self-energy $W_m$ of the ring. It was shown in [2] that in the "local-cylinder" approximation, when the filament can be considered cylindrical over sections small compared with the wavelength of the perturbation, and the external field varies by only a small amount over a distance on the order of the radius $a$ of the filament, the magnetic self-energy is

$$W_m = \frac{(\Phi_0 - \Phi^e)^2}{2L} + \frac{1}{8\pi}\int [(\mathbf{B}^e \cdot \boldsymbol{\tau})^2 - 2\mathbf{B}^{e2}]\, dV + \frac{\Phi_i^2}{8\pi}\int \frac{dl}{S} . \tag{2.1}$$

Here $\Phi_0$ and $\Phi_i$ are the total magnetic field flux through the ring, and the flux of field passing through the transverse cross section of the filament and frozen into the plasma (as the result of ideal conductivity these fluxes are conserved), $\Phi^e$ is the flux of external field $\mathbf{B}^e$ passing through the perturbed ring, L is the self-induction coefficient of the ring for the longitudinal current, $\tau$ is a unit vector tangential to the axial line of the filament, and S is the area of transverse cross section.

In the case under consideration the components of the external magnetic field $\mathbf{B}^e$ in the neighborhood of the ring in equilibrium can be expressed in the following form with an accuracy to quadratic terms in z and r − b:

$$B_r^e = zG(t), \qquad B_\varphi^e = \frac{b}{r} B_e(t), \qquad B_z^e = B_b(t) + (r - b)G(t)$$
$$(G = G_b + G_q = (\partial B_z^e / \partial r)_{r=b,\ z=0}) . \tag{2.2}$$

Here $B_e$ and $B_b$ are the values of the longitudinal and transverse fields on the circle r = b, z = 0; $G_b$ and $G_q$ are the gradients of the basic and stabilizing transverse fields in the neighborhood of the equilibrium filament.

We assume that $B_b$, $G_b$, and the longitudinal current I, induced in the ring by the basic transverse field, are functions of time as $f(t) = f_0 + f_1 \cos \omega t$, while for the case in which both components are nonzero they are associated by the relation

$$I_1 / I_0 = B_{b1} / B_{b0} = G_{b1} / G_{b0} . \tag{2.3}$$

This condition allows the equations of motions of the ring to be considerably simplified.

We omit the cumbersome intermediate calculations and give the final expression for the magnetic self-energy $W_m$ of the ring, obtained from formula (2.1), taking (1.2), (2.2), (2.3), and the expression for L found in [1] into account

$$\begin{aligned} W_m = W_m^{(0)} + V^{(0)} \Big\{ & 2(p_a + p_e - p_i)\alpha_0 + [(K-2)p_a - p_e + p_i]\varepsilon_0 \\ & - \frac{2(l+4-K)}{l} p_a \alpha_0 \varepsilon_0 + \frac{4}{l} p_a \alpha_0^2 + (p_e + 3p_i - p_a)\langle \alpha^2 \rangle \\ & + \frac{(l+4-K)^2 - 4l}{4l} p_a \varepsilon_0^2 + \left[\frac{1}{2}(l+K)p_a + p_e\right]\langle \varepsilon^2 \rangle + 2(p_a - p_e - p_i)\langle \alpha\varepsilon \rangle \\ & - \frac{1}{2} p_a \sum_{n=1}^{\infty} n^2 [(\Lambda - g_r(n))\varepsilon_n^2 + (\Lambda - g_z(n))\delta_n^2] \\ & + \frac{1}{2}(3p_e + p_i)\langle \varepsilon' + \delta' \rangle + \frac{aGB_a}{4\pi}\left(\frac{b}{a}\right)^2 \langle \varepsilon^2 - \delta^2 \rangle - \frac{B_e B_a}{2\pi}\left[\frac{b}{a}\langle \varepsilon\delta' \rangle + \frac{1}{4}(l+K)\frac{a}{b}\langle \alpha\delta' \rangle\right]\Big\} . \end{aligned} \tag{2.4}$$

Here

$$
\begin{gathered}
p_e = B_e{}^2/8\pi, \quad p_i = B_i^2/8\pi \quad (B_i = \Phi_i/\pi a^2), \quad p_a = B_a{}^2/8\pi \quad (B_a = 2I^{(0)}/ca) \\
K = 4b\,B_b/aB_a - l, \qquad l = 2(\Lambda - 2), \qquad \Lambda = \ln(8b/a) \\
g_r(n) = 2\left(1 - \frac{1}{4\pi^2}\right)\sum_{i=1}^{n}\frac{1}{2i-1} + \frac{1}{2} - \frac{1}{n^2}, \quad g_z(n) = 2\left(1 - \frac{3}{4n^2}\right)\sum_{i=1}^{n}\frac{2}{2i-1} + \frac{1}{2}.
\end{gathered} \tag{2.5}
$$

We note that in view of (2.3), the parameter K defined by equations (2.5) is constant. The ratio $B_b/B_a$ is taken to be positive when the directions of the fields $B_b$ and $B_a$ are the same on the outer side of the ring.

In deriving (2.4) it was assumed that $B_a \gg B_b$, $a$G. A stricter treatment (compared with the local-cylinder approximation assumed here) leads to relatively small corrections on the order $a$/b in the expression for $W_m$.

In accordance with (0.1)-(0.3) the magnetic energy $W_m$ of (2.4), together with the expressions for T from (1.7) and U from (1.8), gives the generalized potential energy W and the Routh function R of the perturbed plasma ring.

When the constant terms in the coefficients for $\alpha_0$ and $\varepsilon_0$ in the potential energy W = U + $W_m$ are set equal to zero, we obtain the equilibrium equations of the ring with respect to its smaller and larger radii.

$$
\bar{p}_a + \bar{p}_e - p_i - p_0 = 0, \qquad (K-2)\,\bar{p}_a - \bar{p}_e + p_i - p_0 = 0. \tag{2.6$_0$}
$$

Here and in what follows an overscore above a symbol denotes an average with respect to time. Strictly speaking, in the case of high-frequency fields when the radii of the ring perform forced oscillations, Eqs. (2.6) express the fact that these oscillations occur around the values $a$ and b respectively.

3. The results obtained above enable us, in principle, to investigate equilibrium and stability of a plasma ring for various stabilization regimes employing quadrupole and longitudinal fields. However, because of the coupling between the separate perturbation modes a strict analytic investigation of stability is a rather complicated problem.

The situation is considerably simplified in the limiting case of an infinite straight filament; in particular, the coupling between hosepipe and constrictive perturbations disappears. The results of investigating the stability of a straight filament should also be applicable to an annular filament which is not heavily toroidal, at least for perturbations with n ≫ 1.

To make the limiting transition to a straight filament we set x = b$\varepsilon$, z = b$\delta$, $\rho = a\alpha$, and we let b and n tend to infinity while keeping the ratio n/b = $\kappa$ finite. We obtain the following expressions for the instantaneous kinetic $T_1$ and potential $W_1$ energies:

$$
T_1 = \frac{1}{2}\pi a^2\sigma_0\left\{\frac{1}{2}\dot{\rho}_0^2 + \dot{x}_0^2 + \dot{z}_0^2 + \sum_{k\neq 0}\left[2\frac{\dot{\rho}_k^2}{(ka)^2} + \frac{1}{2}(\dot{x}_k^2 + \dot{z}_k^2)\right]\right\} \tag{3.1}
$$

$$
\begin{aligned}
W_1 = W_1^{(0)} &+ \pi\{2(p_{c\omega} + p_{e\omega})\,a\rho_0 + 2\gamma p_0\rho_0^2 + (p_e + 3p_i - p_a - p_0)\langle\rho^2\rangle \\
&+ \frac{1}{2}\sum_{k\neq 0}(ka)^2\left[p_e + p_i - p_a \ln\frac{2}{\beta ka} + \frac{1}{2}(p_{a\omega} + p_{e\omega})\right](x_k^2 + z_k^2) \\
&+ \frac{aG_q B_a}{4\pi}\langle x^2 - z^2\rangle + \frac{B_e B_a}{4\pi}\sum_{k\neq 0} ka\,(x_{ks}z_{kc} - x_{kc}z_{ks})\}.
\end{aligned} \tag{3.2}
$$

Here $\rho$, x and z are expanded in series of the form of (1.3), in which ky appears in place of n$\varphi$. Here y is the coordinate measured along the axis of the unperturbed filament, $\sigma_0$ is the equilibrium density of the plasma, $\ln\beta$ = 0.577 is Euler's constant, and the subscript $\omega$ denotes the high-frequency components of the corresponding quantities.

We shall treat the stability of the filament relative to kink perturbations in greater detail. The kinetic energy (3.1) for kinks assumes the form

$$
T_1 = \frac{1}{2}\pi a^2\sigma_0\sum_{k\neq 0}(\dot{x}_k^2 + \dot{z}_k^2). \tag{3.3}
$$

It is convenient to represent the increment $W_1^{(1)}$ in potential energy (3.2) in the form

$$W_1^{(1)} = \frac{1}{2}\pi\bar{p}_a \sum_{k\neq 0} [C_k(x_k^2 + z_k^2) + D_q(x_k^2 - z_k^2) + 2D_{ek}(x_{ks}z_{kc} - x_{kc}z_{ks})]. \tag{3.4}$$

Here

$$C_k = (ka)^2 [p_e^* + p_i^* - p_a^* \ln(2/\beta ka) + {}^1/_2 (p_{a\omega}^* + p_{e\omega}^*)]$$
$$D_q = aG_qB_a/4\pi\bar{p}_a, \qquad D_{ek} = kaB_eB_a/4\pi\bar{p}_a$$

where the asterisk denotes ratios of the corresponding quantities with respect to $\bar{P}_a$. In what follows the subscript k will be omitted where this does not complicate the issue.

We now pass to the new coordinates u and v defined by the formulas

$$\sqrt{2}x_{kc} = u_{kc} + v_{ks}, \qquad \sqrt{2}z_{kc} = v_{kc} + u_{ks}$$
$$\sqrt{2}x_{ks} = u_{ks} - v_{kc}, \qquad \sqrt{2}z_{ks} = v_{ks} - u_{kc}.$$

In this case

$$T_1 = \frac{1}{4}\pi a^2\sigma_0 \sum_{k\neq 0} (u_k^2 + v_k^2) \tag{3.5}$$

$$W_1^{(1)} = \frac{1}{2}\pi\bar{p}_a \sum_{k\neq 0} [C_k(u_k^2 + v_k^2) + D_{ek}(u_k^2 - v_k^2) + 2D_q(u_{ks}v_{kc} - u_{kc}v_{ks})] \tag{3.6}$$

$$(u_k^2 = u_{kc}^2 + u_{ks}^2, \qquad v_k^2 = v_{kc}^2 + v_{ks}^2).$$

Comparing equations (3.3) and (3.4) on the one hand and (3.5) and (3.6) on the other hand we note that for a system with a single quadrupole field the normal oscillations are flat coils in the xz plane when $D_e = 0$, while for a system with a single longitudinal field the normal oscillations are in u and v when $D_q = 0$. It can easily be shown that the latter are three-dimensional helixes corresponding to perturbations with an azimuthal wave number $m = \pm 1$ in the ordinary magnetohydrodynamic approach.

The formal similarity of expressions (3.4) and (3.6) for $W_1^{(1)}$ is an indication of the specific similarity which exists between the two types of stabilization considered, one using a quadrupole field and the other a longitudinal field. However, the difference in the coefficients does not allow the results of investigating one type of stabilization to be transferred directly to the other.

The equations of motion in both particular cases in which there is either only a quadrupole or only a longitudinal field can be written in the form

$$\ddot{\xi} + \Omega^2(C \pm D)\xi = 0 \qquad (\Omega^2 = 2\bar{p}_a / a^2\sigma_0). \tag{3.7}$$

Here $\Omega$ is some characteristic frequency of the system; the plus sign is for the z and y coordinates and the minus sign is for the x and u coordinates.

In the case of constant fields the condition for the solution of Eq. (3.7) to be stable is clearly $C - D > 0$. It immediately follows from this that a filament with a constant current is unstable in the absence of a stabilizing field.

In fact $D = 0$ in this case, and $C = -(ka)^2\ln(2/\beta ka) < 0$ for sufficiently long wave perturbations. A constant quadrupole field does not stabilize the filament (in the approximation assumed here it does not even appear in the expression for the coefficient C). At the same time a large constant longitudinal field can ensure stability; taking $B_e = B_i \gg B_a$ we arrive at the familiar Shafranov-Kruskal criterion

$$B_e > B_a / ka. \tag{3.8}$$

If the current in the filament or the external field changes with a high frequency, the possibilities of stabilization are considerably greater, stabilization by means of a quadrupole field turns out to be possible, and criterion (3.8) can be relaxed considerably in the case of a longitudinal field.

Different variants of dynamic stabilization of a fine filament, with a quadrupole field, are treated in [1.3]. The problem of the dynamic stabilization of a thin plasma filament by a longitudinal magnetic field is solved in a similar manner with the help of the equations given above. The results thus obtained actually coincide with those obtained by the ordinary magnetohydrodynamic method in papers [4,5]. The theory developed above enables us to investigate the general case also, when both stabilizing fields, a longitudinal and a quadrupole, are present. An investigation of this type is carried out in the next section for one of the variants of combined dynamic stabilization, when a high-frequency current is excited in the filament and the stabilizing fields are constant.

4 It is well known (see, for example, [1, 3-5]) that long-wave kinks in a current-carrying plasma filament are better stabilized by a quadrupole magnetic field, while short-wave kinks are better stabilized by a longitudinal field.

It is natural to assume that by combining a quadrupole field with a longitudinal field, we should be able to extend the range of perturbations which can be stabilized, or alternatively decrease the strength of the stabilizing fields. The most convenient system is one in which a filament with an alternating current $I^{(0)} = I_1 \cos \omega t$ is stabilized by constant quadrupole and longitudinal fields. We then have for the coefficients of the potential energy

$$C = C_0 + C_2 \cos 2\omega t, \quad D_q = D_{q1} \cos \omega t, \quad D_e = D_{e1} \cos \omega t$$

where $C_0$, $C_2$, $D_{q1}$ and $D_{e1}$ are constants. When $D_q$ and $D_e$ are harmonic functions of time, the coordinates x and z (or u and v) appear entirely symmetrically in the expression for $W_1^{(1)}$ as well as in the expression for $T_1$. It is well known (see, for example, [6]) that in this case the system exhibits so called "difference" resonance of coupling, in which energy is pumped from one partial oscillation to another, while the total energy remains constant, and consequently the amplitude of each oscillation cannot increase without limit. Thus coupling in this case does not destroy the stability of the system if the filament is stable with one of the stabilizing fields; it remains stable on the introduction of the other field. This means that the stability criteria obtained for a filament with an alternating current in quadrupole [3] and longitudinal [5] fields are also sufficient stability conditions in the general case of combined stabilization.

Allowing for both stabilizing fields simultaneously should, clearly lead to a less restrictive stability criterion than in [3] or [5]. It is, however, exceedingly difficult to carry out this calculation exactly by investigating a system of coupled equations with periodic coefficients. Keeping in mind what has been already said, we shall confine ourselves to considering motion averaged over the rapid oscillations, assuming that the frequency $\omega$ is much greater than the characteristic frequency of oscillation or the instability increment of the system in the absence of oscillation.

The effective potential energy of average motion is defined [7] by the equation

$$W_{eff} = \overline{W} + \frac{1}{2} \sum_j \frac{1}{\omega_j^2} \sum_{i,k} a_{ik}^{-1} \overline{\frac{\partial W_{j\omega}}{\partial \xi_i} \frac{\partial W_{j\omega}}{\partial \xi_k}} .$$

Here $W_{j\omega}$ is the component of potential energy; oscillating with frequency $\omega_j$. $a_{ik}^{-1}$ are elements of a matrix which is the inverse of the matrix of the coefficients in the kinetic energy of this system. In the present case we obtain, after some calculations,

$$W_{1eff}^{(1)} = \frac{1}{2} \pi \overline{p}_a \sum_{k \neq 0} (ka)^2 \left\{ p_e^* + p_i^* - \ln \frac{2}{\beta ka} + \frac{4}{\nu^2} \left[ \frac{p_q^*}{(ka)^2} + p_e^* + \frac{1}{32} (ka)^2 \left( \ln \frac{2}{\beta ka} - \frac{1}{2} \right)^2 \right] \right\} (x_k^2 + z_k^2) . \quad (4.1)$$

where $\nu = \omega/\Omega$, and $p_q = (aG_q)^2/8\pi$. An important characteristic of expression (4.1) is the absence of cross terms. This means that in smooth motion the separate harmonics in the perturbation are not coupled to each other, i.e., they are normal modes of the system.

The condition for the method of averaging to be applicable has the form

$$\nu^2 \gg (ka)^2 \left| p_e^* + p_i^* - \ln \frac{2}{\beta ka} \right|, \; p_q^{*1/2} . \quad (4.2)$$

If, taking (4.2) into account, we neglect the last term in brackets in (4.1) the stability criterion for a filament relative to kinks can be written in the form

$$p_q^*/(ka)^2 + (1 + {}^1\!/_4\nu^2)\, p_e^* + {}^1\!/_4\nu^2 p_i^* > {}^1\!/_4\nu^2 \ln(2/\beta ka). \tag{4.3}$$

Setting $p_q^* = 0$ or $p_e^* = 0$ we obtain the stability conditions for a filament in the particular cases in which there is only a longitudinal or only a quadrupole stabilizing field.

Figures 1 and 2 illustrate criterion (4.3) for $B_e = B_i$ and $\nu = 2$, satisfying condition (4.2). As is to be expected, the introduction of a longitudinal field allows the quadrupole field to be decreased (or more exactly, its gradient). The greater ka, i.e., the smaller the wavelength of the perturbation (Fig. 1), the more it can be decreased. On the other hand, a quadrupole field allows the longitudinal field to be decreased. The smaller $ka$, i.e., the larger the wavelength of the perturbation, the more it can be decreased (Fig. 2).

As the parameter $\nu$ increases, the stabilizing fields also increase as can be seen from (4.3). The quadrupole field increases much more rapidly than the longitudinal field.

We shall now briefly dwell on the stability of the filament relative to constrictive-type perturbations. It can easily be seen that formally the same equations are obtained for perturbations of this type as in [3] for the case $B_e = 0$. If $\nu \gtrless 1$, the stability condition for the constrictions themselves ($k \neq 0$) virtually coincides with the well-known criterion for the stabilization of a frozen-in magnetic field

$$p_i^* > {}^1\!/_2 \tag{4.4}$$

When (4.4) is satisfied, oscillations of the filament radius also turn out to be stable if we take into account the bandwidths of parametric resonance which are still narrower than those in [3]. The narrowing of the resonance regions comes about as the result of an increase in the rigidity of the system because of the constant external magnetic field $B_e$.

If the relative "cost" of quadrupole and longitudinal fields is known, by using diagrams similar to those given in Figs. 1 and 2 and taking criterion (4.4) into account, we can select the optimum operating regime for the system.

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